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Modal Ring Method for the Scattering of Sound

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SUMMARY

The modal element method for acoustic scattering can be simplified when the scattering body is rigid. In this simplified method, called the modal ring method, the scattering body is represented by a ring of triangular finite elements forming the outer surface. The acoustic pressure is calculated at the element nodes. The pressure in the infinite computational region surrounding the body is represented analytically by an eigenfunction expansion. The two solution forms are coupled by the continuity of pressure and velocity on the body surface.

The modal ring method effectively reduces the two-dimensional scattering problem to a one-dimensional problem capable of handling very high frequency scattering. In contrast to the boundary element method or the method of moments, which perform a similar reduction in problem dimension, the modal line method has the added advantage of having a highly banded solution matrix requiring considerably less computer storage. The method shows excellent agreement with analytic results for scattering from rigid circular cylinders over a wide frequency range ($1 \leq ka \leq 100$) in the near and far fields.

INTRODUCTION

The modal element method, which couples finite elements and eigenfunction expansions, has been employed in electromagnetic and acoustic scattering and duct transmission problems. The primary reasons for employing this technique are (1) to describe accurately the radiation boundary condition at the computational boundary and (2) to reduce the size of the numerical grid. This hybrid steady state method has been given various titles, such as the unimoment method, the transfinite element method, and the modal element method. In electromagnetics, Chang and Mei (1976) and Lee and Cendes (1987) applied the method to scattering from dielectric cylinders while Baumeister (1991) applied the method to electromagnetic propagation in ducts. In acoustics, Astley and Eversman (1981) employed the method in duct propagation problems while Baumeister and Kreider (1993) have applied the method to acoustic scattering problems.

The purpose of this paper is to develop and study an extension of the modal element method for scattering from rigid bodies. The goal is to minimize the domain in which finite elements are employed. This approach is called the modal ring method and was briefly introduced by Baumeister and Kreider (1993).

The modal ring method can effectively reduce a two-dimensional scattering problem to a one-dimensional problem by employing a ring of nodes along the surface of the rigid scattering body. Herein, a variety of grid configurations and parameters are explored. The method is validated by applying it to the simple case of scattering from a hard circular cylinder, for which analytic solutions are easily obtained.

NOMENCLATURE

A_m^+	modal amplitude of wave moving in +r direction away from origin
a	dimensionless circular cylinder radius
e_r	relative error
$H_m^{(1)}$	Hankel function of the first kind
i	$\sqrt{-1}$
k	wave number
m	mode number
M_{coef}	number of modal coefficients used in eigenfunction expansion
p	dimensionless perturbation acoustic pressure
r	dimensionless radial coordinate
ϵ	dimensionless complex acoustic permittivity
Θ	angle between position vector and x axis
μ	dimensionless complex acoustic permeability
ω	dimensionless frequency

METHOD OF ANALYSIS

This study is concerned with computing the acoustic scattering by a two-dimensional rigid body of an impinging plane wave traveling in the +x direction. The spatial domain is divided into two subdomains, the finite element domain, which contains the body, and the homogeneous domain, which surrounds the body and extends to infinity. Linear triangular elements are used in the finite element

domain to calculate the pressure at the nodes. In the homogeneous domain, an eigenfunction expansion represents the acoustic pressure. The two solution forms are coupled by imposing continuity on the pressure and velocity at the interface between the two subdomains. This coupling results in a single matrix equation in which the eigenfunction coefficients and the pressures at the finite element nodes are calculated simultaneously, yielding a global representation of the acoustic field.

GEOMETRICAL MODEL

For penetrable bodies, the internal pressure field must be known to determine the scattered field. The modal element method (Baumeister and Kreider, 1993) uses a finite element grid inside the body to calculate

the internal pressure field. For rigid bodies, studied here, the field does not penetrate the body, so the grid can be greatly reduced, yielding a variation of the method called the modal ring method.

In the modal ring method, the scattering body is represented by a thin ring of finite elements following the body's contour. Figure 1 shows three possible configurations. The grid in figure 1(a) consists of a single row of boundary nodes with a central node. The grid in figure 1(b) has a ring of elements on the body's surface and a central node, so the body is filled with elements while the grid in figure 1(c) has the same ring of elements but excludes the central region, where the field vanishes. The rigid body is simulated numerically, through an impedance mismatch induced by setting $\epsilon = 1 - 10^{19}i$ and $\mu = 1$ at each internal finite element node. This feature allows greater flexibility in the numerical implementation of the method, because penetrable or coated rigid bodies may be studied with only slight modifications to the computer code, mainly in grid generation. For penetrable bodies, the internal grid is more extensive, while for coated bodies, several rings in the coating region may be needed.

GOVERNING EQUATIONS

Acoustic propagation in two-dimensional space can be modeled by the continuity, momentum, and state linearized gas dynamic equations in the absence of flow. For harmonic pressure propagation in an inhomogeneous material, the following dimensionless wave equation applies:

$$\frac{\partial}{\partial x} \left(\frac{1}{\epsilon} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{1}{\epsilon} \frac{\partial p}{\partial y} \right) + \omega^2 \mu p = 0 \quad (1)$$

The harmonic time dependence $e^{-i\omega t}$ has been factored out. In analogy with electromagnetic scattering, ϵ is the acoustic "permittivity," μ is the acoustic "permeability" and ω is the dimensionless frequency. The wave number is

$$k = \sqrt{\omega^2 \mu \epsilon} \quad (2)$$

At the interface between the finite element region and the analytic region, continuity is imposed on the pressure and velocity. The radiation boundary condition at infinity is automatically satisfied by the eigenfunction expansion introduced in the next section.

ANALYTIC SOLUTION

In the homogeneous domain, an exact eigenfunction expansion can be derived from equation (1) by separation of variables. An incident plane wave travelling in the +x direction that strikes a symmetric two-dimensional scatterer generates a pressure field approximated by

$$p = p^i + p^s = e^{ikx} + \sum_{m=0}^{M_{\text{coef}}-1} A_m^+ H_m^{(1)}(kr) \cos(m\theta) \quad (3)$$

p^i is the incident plane wave; p^s is the scattered wave. The modal coefficients A_m^+ are unknowns to be determined. Formulas to estimate the number of modes needed for convergence can be found in Baumeister and Kreider (1993).

FINITE ELEMENT SOLUTION

The finite element domain is divided into triangular elements with unknown acoustic pressure at the nodes. It is assumed that all material properties are constant in each element. Details on the numerical setup and solution can be found in Baumeister and Kreider (1993).

RESULTS AND COMPARISONS

The modal ring method may be implemented using various grid systems and parameter values for ring thickness and the number of nodes. In order to study the influence of these parameters on numerical results, the following simple standard scattering problem is considered: a unit plane wave, incident from the left with a frequency ranging from $k = 0.1$ to $k = 100$, strikes a rigid circular cylinder of dimensionless radius $r = 1$ oriented with its axis normal to the propagation direction. In the homogeneous region surrounding the scatterer, $\varepsilon = 1$ and $\mu = 1$, while within the scatterer, $\varepsilon = 1 \cdot 10^{19}$ and $\mu = 1$. The number of modes M_{coef} used in the eigenfunction expansion in equation (3) is determined by formulas in Baumeister and Kreider (1993, eqs. (26) and (27)). All of the plots presented here are generated using equation (3) with the calculated A_m^+ values.

Number of Nodes

As the frequency of the incident wave increases, more finite element nodes are required to resolve the increasing oscillations in both the pressure and intensity fields. The oscillations in this case are represented by the cosine terms in equation (3), so the number of boundary nodes depends on the number of eigenfunction modes used. Experience has shown that setting the number of boundary nodes to $12M_{\text{coef}}$ suffices; this condition is used in the examples shown below.

Grid Systems

Grid 1(a), with its single ring of boundary nodes, is attractive because it uses the least number of nodes for a given problem. However, it directly links the boundary of the body with the center node. This inflexibility yields poor numerical results—relative errors up to 30 percent for low frequencies. This grid should not be used for any scattering problem.

In contrast, grids 1(b) and (c) consist of a ring of elements following the body's surface. Grid 1(b) contains a central node, while grid 1(c) excludes the central region. When using grid 1(c), the pressure gradient along the inner element boundaries is set to zero. Numerical results for these grids are virtually identical, indicating that the presence of the central node is not critical. Figure 2 shows the pressure amplitude at radius $r = 3.18$ with frequency $ka = 25$ using grid 1(b). Here, $M_{\text{coef}} = 40$ modes are required for the scattered field in equation (3), so 480 boundary nodes are used. Figure 3 shows the pressure amplitude at radius $r = 3.18$ with frequency $ka = 50$ using grid 1(c) with 804 nodes ($M_{\text{coef}} = 67$). In both figures, the numerical results (squares) are in excellent agreement with the analytic solutions (solid lines).

Both grids were tested with frequencies up to $ka = 100$ with accuracy comparable to that seen in figures 2 and 3. The plots are not included here because the high degree of oscillation in the graphs makes visual inspection difficult.

Ring Thickness

The thickness of the ring of elements surrounding the body is very significant. For a cylinder of radius 1, element thicknesses ranging from 0.05 to 0.001 were considered. Figure 4 shows results for incident frequency $k = 25$. Here, $M_{\text{coef}} = 40$, so 480 boundary nodes are used. Grid 1(b) is used. For element thickness 0.05 (fig. 4(a)), results are poor. For element thickness 0.01 (fig. 4(b)), results are better, but there is still some deviation from the exact solution. Reducing the element thickness to 0.001 (fig. 4(c)) yields excellent results.

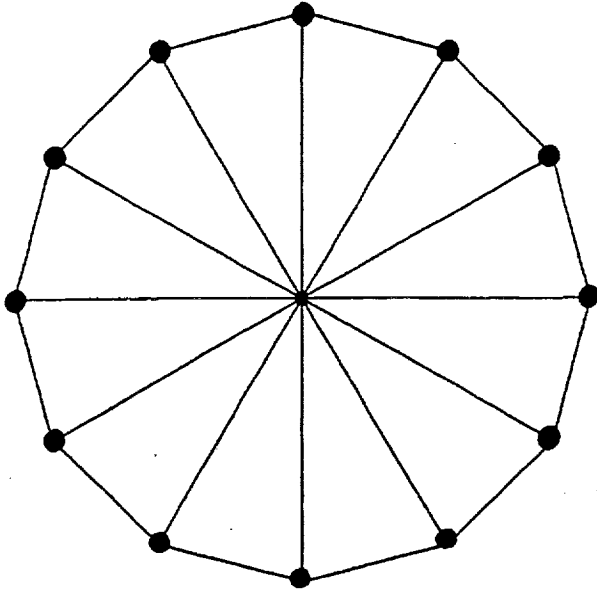
CONCLUDING REMARKS

The modal ring method for acoustic scattering from a two-dimensional rigid body is presented. The acoustic pressure field is represented by a finite element solution on a thin ring of elements on the surface of the scattering body, and by an eigenfunction expansion outside the body. The two representations are coupled by the continuity of pressure and velocity across the interface between the two subdomains, and are calculated simultaneously from a single matrix equation. This matrix is highly banded because the usual two-dimensional finite element grid is replaced by an essentially one-dimensional grid. This reduction in computational resources is the main benefit of the modal ring method, but it may be used only when the scattering body is rigid. However, the method is applicable to problems involving very high or low frequency scattering from rigid bodies of complicated shape. In the validation cases presented, numerical results are in excellent agreement with the corresponding exact solutions.

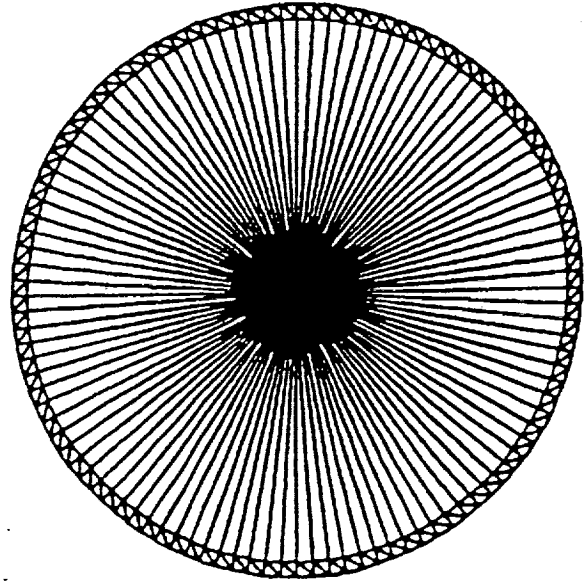
The grid system most suitable for use with the modal ring method contains a very thin ring around the surface of the scattering body, and may or may not include a node at the body's center. If the central node is not included, the pressure gradient along the inner element boundaries is set to zero.

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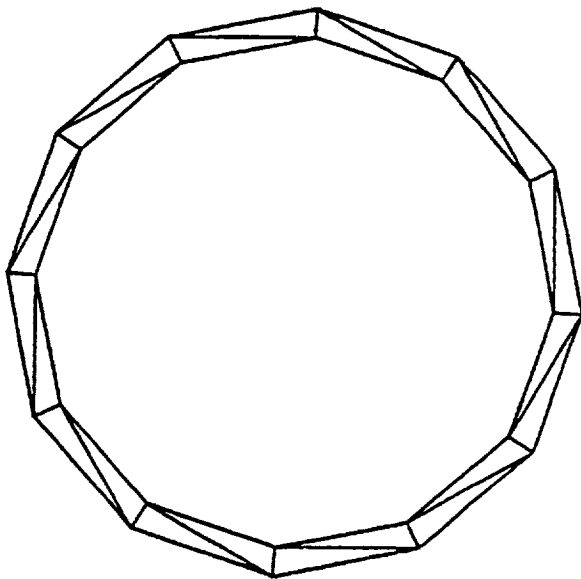
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(a) Line of nodes with central node.



(b) Line of elements with central node.



(c) Line of elements without central node.

Figure 1.—Finite element ring grid system.

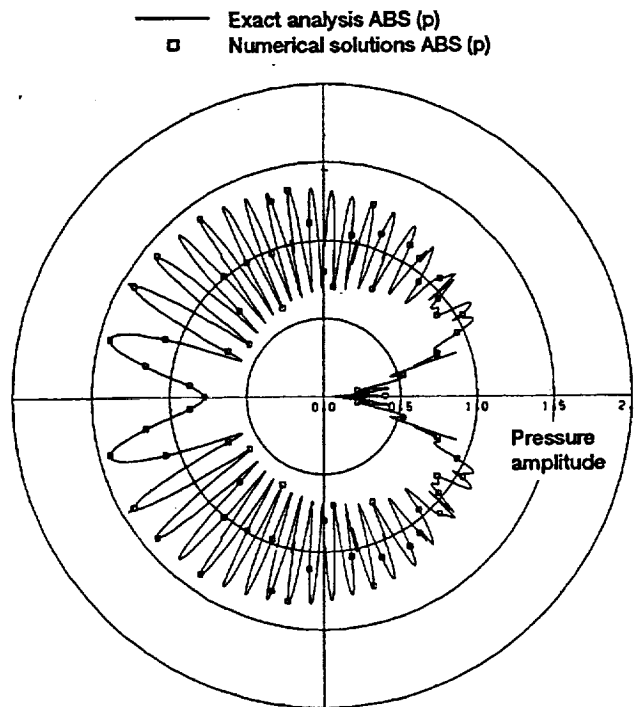


Figure 2.—Polar Plot of the total acoustic pressure around a rigid cylinder modeled by a single line of symmetrical elements on boundary with central node for configuration figure 1 (b) ($ka = 25.0$, $a = 1$, number of nodes = 961, number of elements = 1440, $M_{coef} = 40$, $\Delta t = 0.001$, $r = 3.18$, $kr = 79.58$).

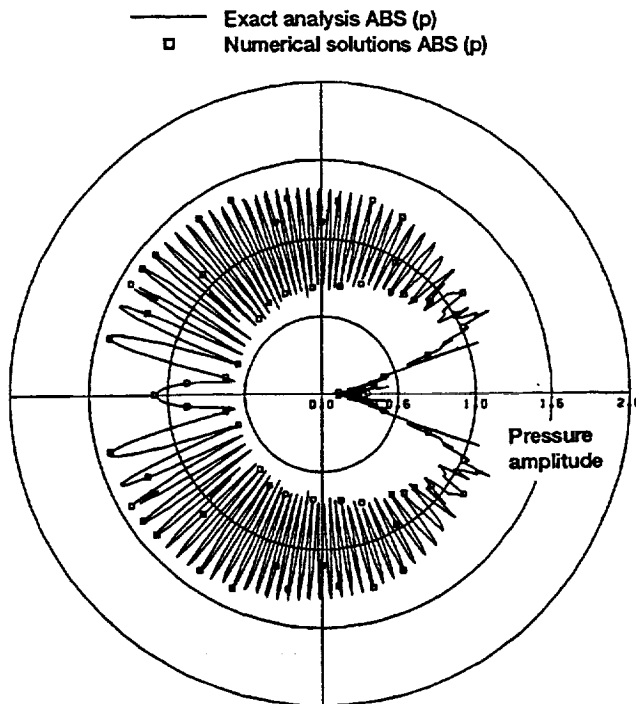
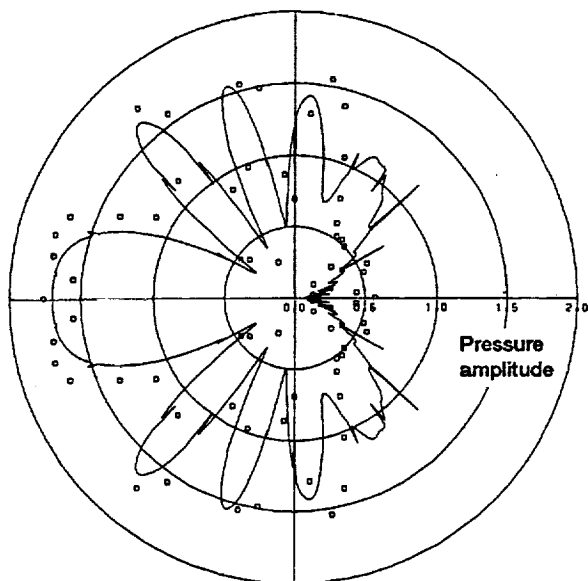
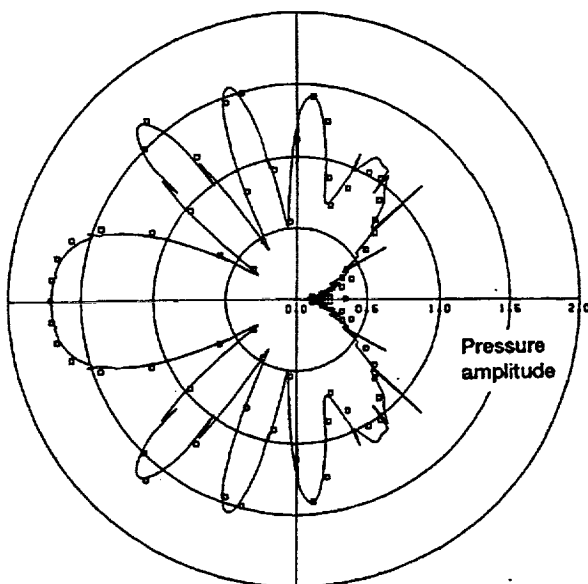


Figure 3.—Polar Plot of the total acoustic pressure around a rigid cylinder modeled by a single line of symmetrical elements on boundary for configuration figure 1(c) ($ka = 50.0$, $a = 1$, number of nodes = 1608, number of elements = 1608, $M_{coef} = 67$, $\Delta t = 0.001$, $r = 3.18$, $kr = 159.16$).

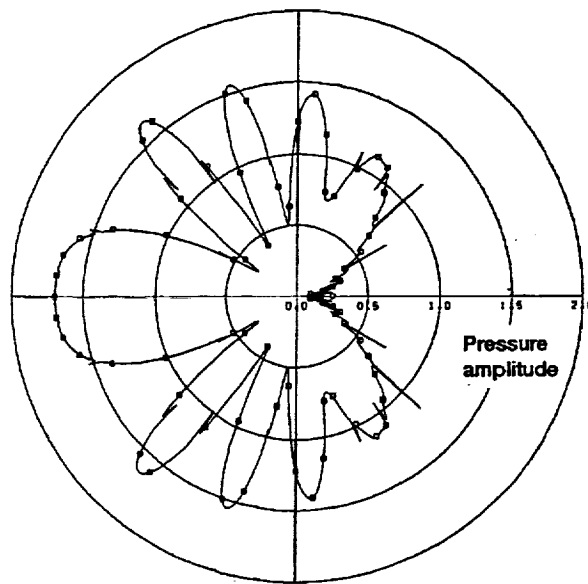


(a) $\Delta t = 0.05$.

— Exact analysis ABS (p)
 □ Numerical solutions ABS (p)



(b) $\Delta t = 0.01$.



(c) $\Delta t = 0.001$.

Figure 4.—Sensitivity of the total acoustic pressure around a solid cylinder to line element thickness Δt ($ka = 25.0$, $a = 1$, $r = 1.5$, number of nodes = 961, number of elements = 1440, $M_{\text{coef}} = 40$, configuration figure 1 (b)).

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